Deep Generative Models

Hongyue Li

Sampling

Sample a random number from uniform [0,1]?

Pseudo-random number generator (PRNG), which is an algorithm that produces a sequence of numbers that *appears random*.

Linear Congruential Generator (LCG)

Parameters:

$$X_0 \text{ (seed)}, \quad a = 22{,}695{,}477, \quad c = 1, \quad m = 2^{31}$$

Step 1 — Generate next integer:

$$X_{n+1} = (aX_n + c) \bmod m$$

Step 2 — Optional normalization:

$$U_{n+1} = \frac{X_{n+1}}{m} \in [0, 1)$$

Step 3 — Repeat:

Use X_{n+1} as the new seed and iterate as many times as needed.

Sample a random number from Normal distribution N(0,1)?

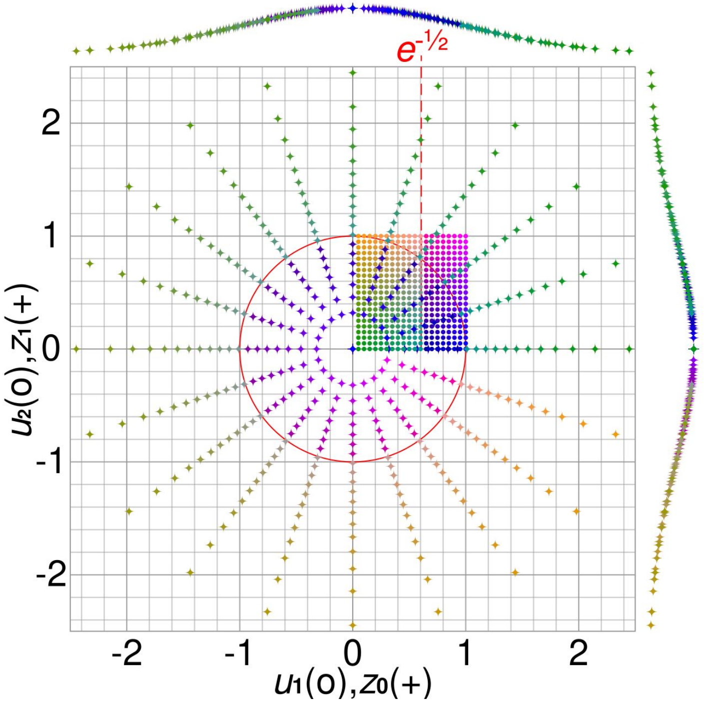
start from uniform distribution, then **transform** it to normal distribution

Box-Muller Transform

$$U_1, U_2 \sim \mathrm{Uniform}(0,1)$$

$$Z_0 = \sqrt{-2\ln U_1}\,\cos(2\pi U_2),$$
$$Z_1 = \sqrt{-2\ln U_1}\,\sin(2\pi U_2)$$

$$Z_0, Z_1 \sim \mathcal{N}(0,1), \quad \text{independent}$$



Sample a random state from Ising model?

For spins $s_i \in \{+1, -1\}$ on graph G with couplings J_{ij} and external fields h_i,

$$P(s) = \frac{1}{Z} \exp\left(\beta \sum_{\langle i,j \rangle} J_{ij} s_i s_j + \beta \sum_i h_i s_i\right),$$

where $\beta = 1/(k_B T)$. Energy $E(s) = -\sum_{\langle i,j \rangle} J_{ij} s_i s_j - \sum_i h_i s_i$.

Gibbs Sampling Steps

1. **Pick a spin** i to update (randomly or in sequence).
2. **Compute the local field** acting on that spin:

$$H_i = \sum_j J_{ij} s_j + h_i.$$

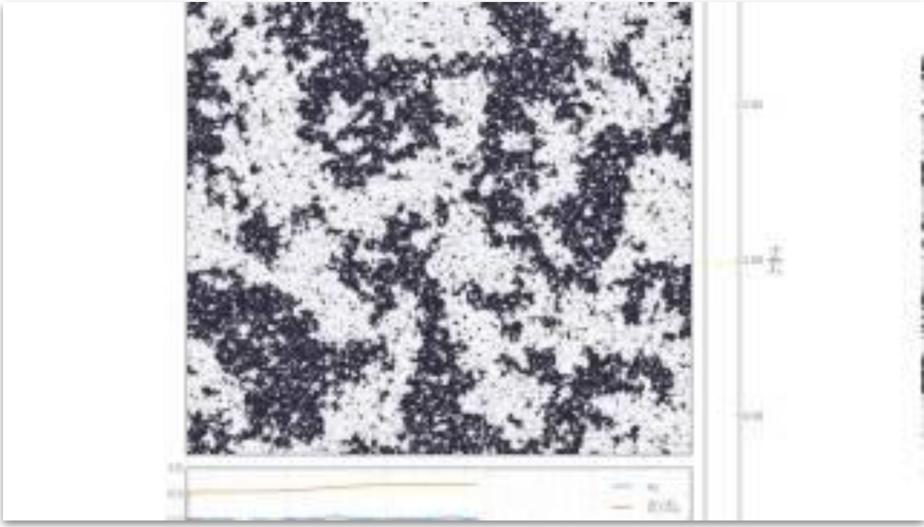
3. **Compute the conditional probability** that spin i is $+1$:

$$P(s_i = +1 \mid s_{\setminus i}) = \frac{e^{\beta H_i}}{e^{\beta H_i} + e^{-\beta H_i}} = \frac{1}{1 + e^{-2\beta H_i}}.$$

4. **Sample** $s_i^{(t+1)}$ from this Bernoulli distribution:

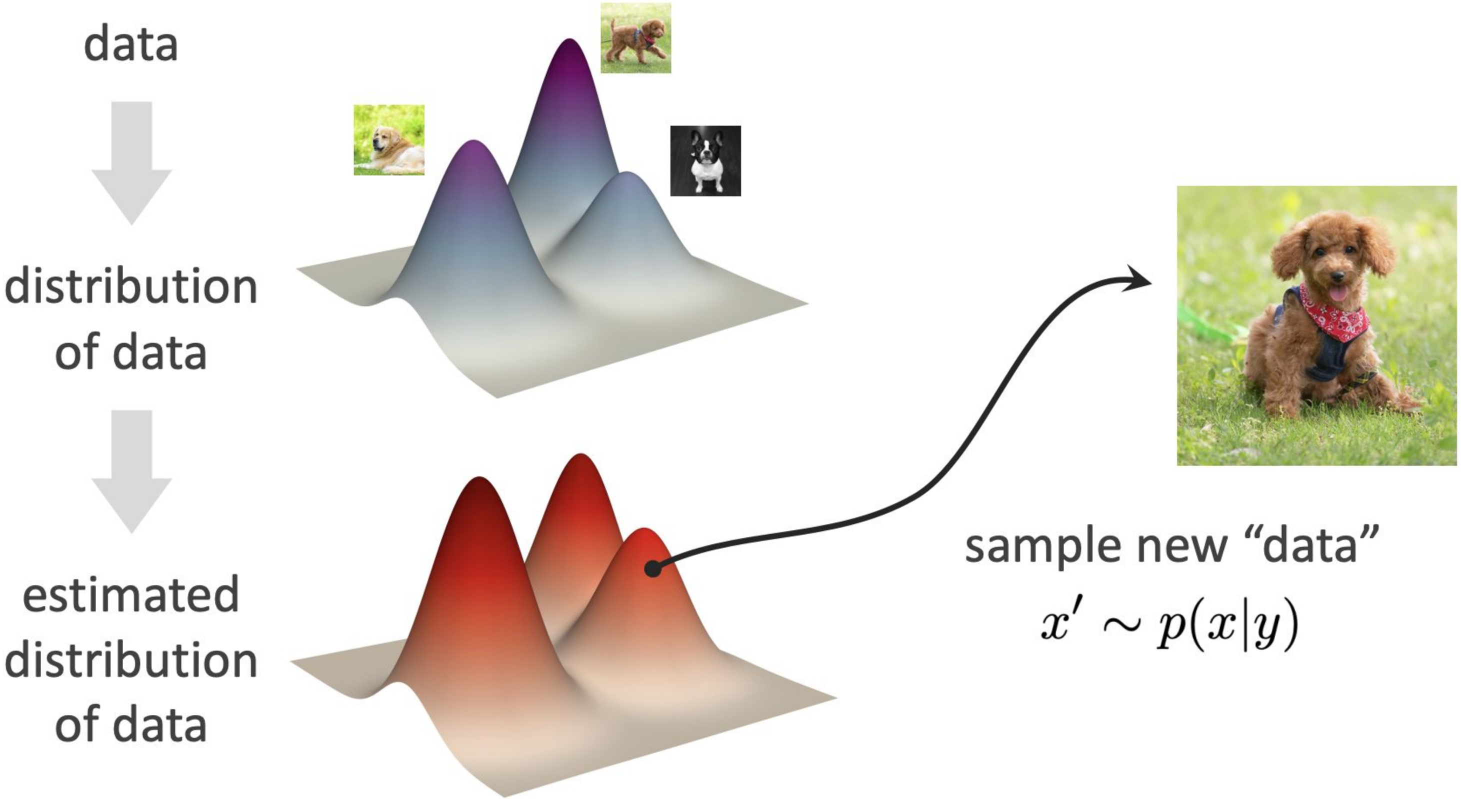
$$s_i^{(t+1)} = \begin{cases} +1 & \text{with probability } p_i = \frac{1}{1+e^{-2\beta H_i}}, \\ -1 & \text{with probability } 1 - p_i. \end{cases}$$

5. **Repeat** steps 1–4 for all spins (one "sweep") to get the next full configuration $s^{(t+1)}$.
6. **Iterate** many sweeps until samples approximate the stationary distribution $P(s)$.

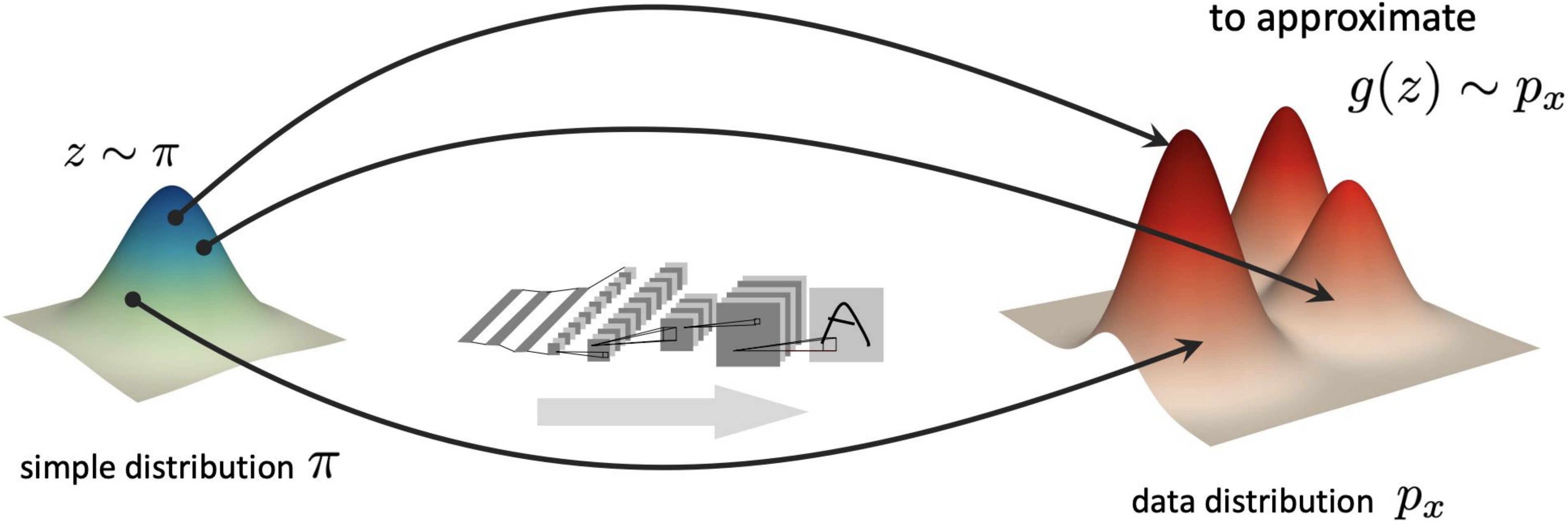


Generative Modeling

How to generate data sampled from some distribution?



Transform a simple distribution to a complex one that approximates the data distribution



Deep Generative Model

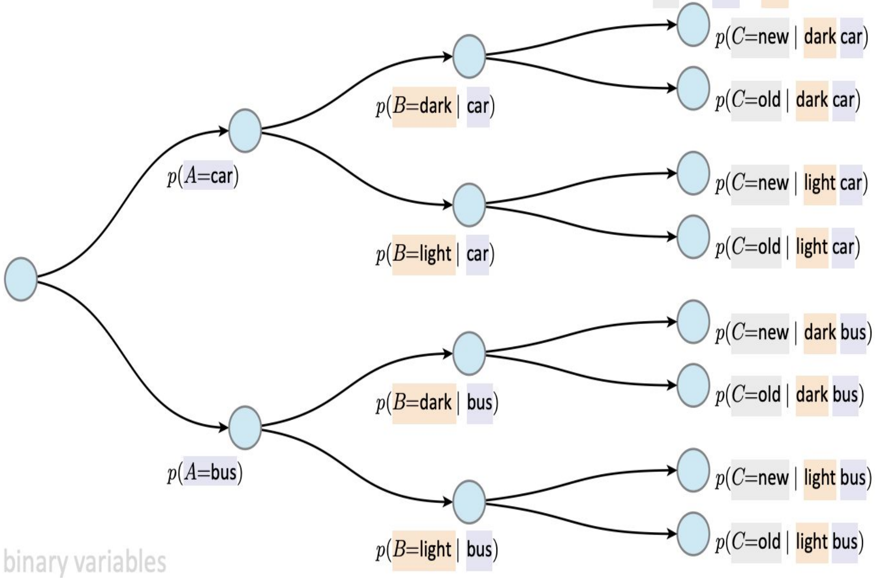
- Modeling & Learning
  - Formulation: frame the problem as probabilistic modeling
  - Representation: deep neural networks to represent data distribution
  - Objective: to measure how good the predicted distribution is
  - Symmetry: decompose complex distributions into simple and tractable ones
- Inference:
  - sampler: to produce new samples
  - probability density estimator (optional)

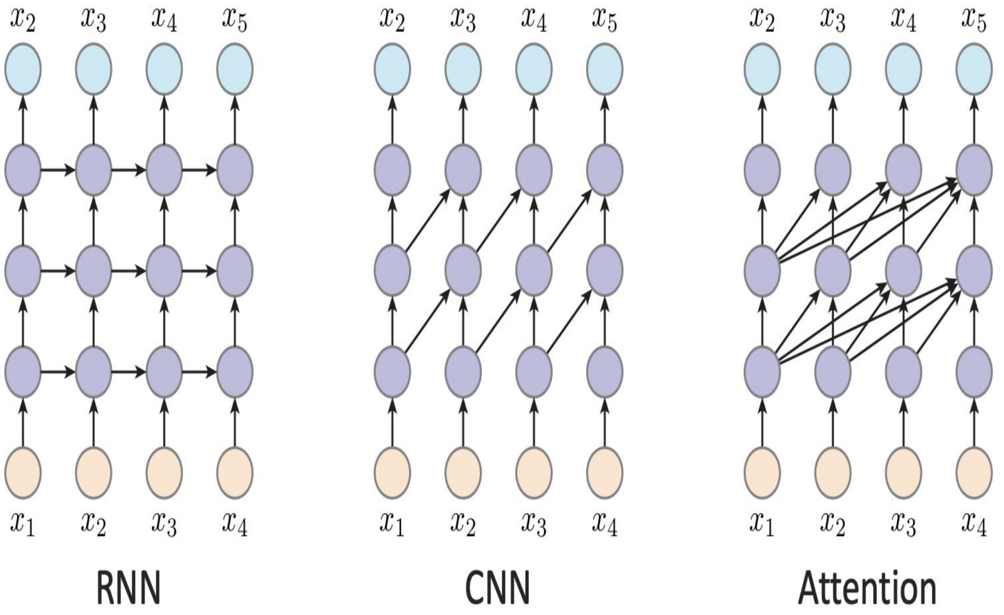
Probabilistic Graphical Model & Autoregressive Model

Formulation: frame the problem as probabilistic modeling

$$p(x_1, x_2, ..., x_n) = p(x_1)p(x_2 \mid x_1)...p(x_n \mid x_1, x_2, ..., x_{n-1})$$

$$p(A, B, C) = p(A)p(B \mid A)p(C \mid A, B)$$

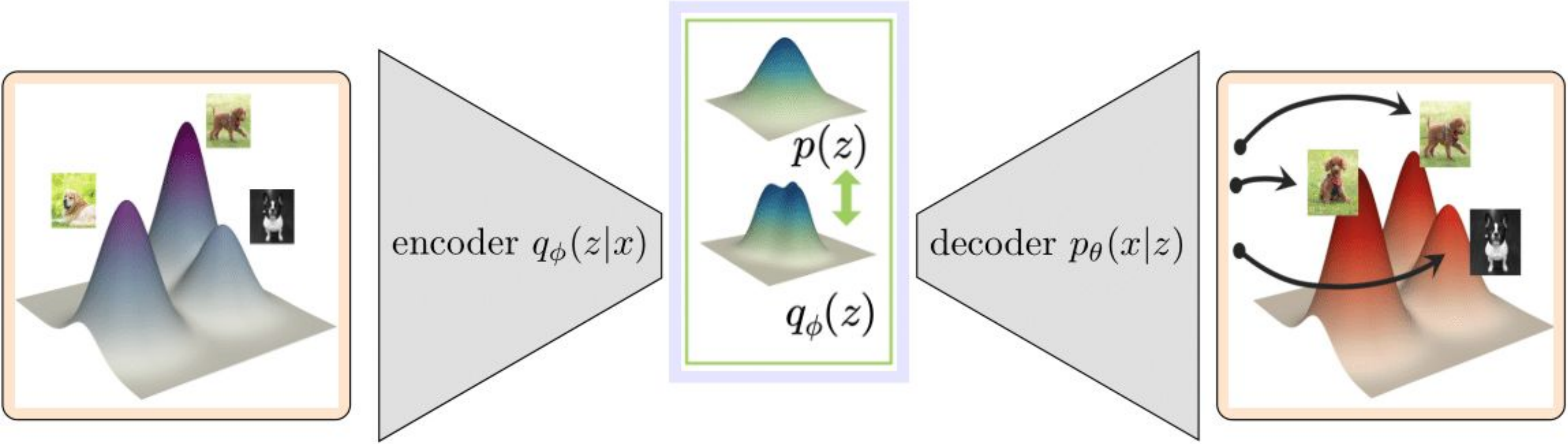




Variational Autoencoder

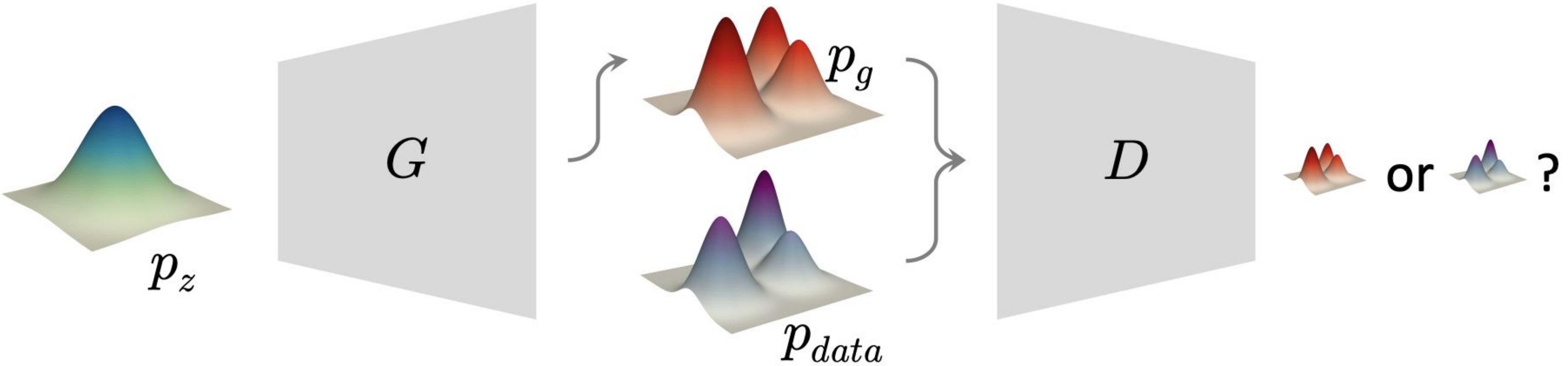
Representation: deep neural networks to represent data distribution

$$\mathcal{L}_{\theta,\phi}(x) = \boxed{-\mathbb{E}_{z\sim q_\phi(z|x)}\Big[\log p_\theta(x|z)\Big]} + \boxed{\mathcal{D}_{\mathrm{KL}}\Big(q_\phi(z|x)||p(z)\Big)}$$



Generative Adversarial Networks

Objective: to measure how good the predicted distribution is

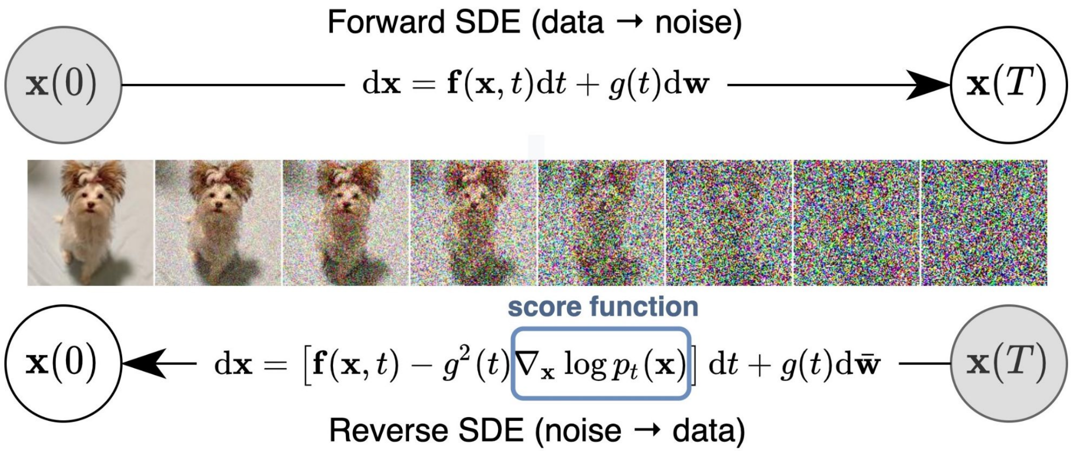


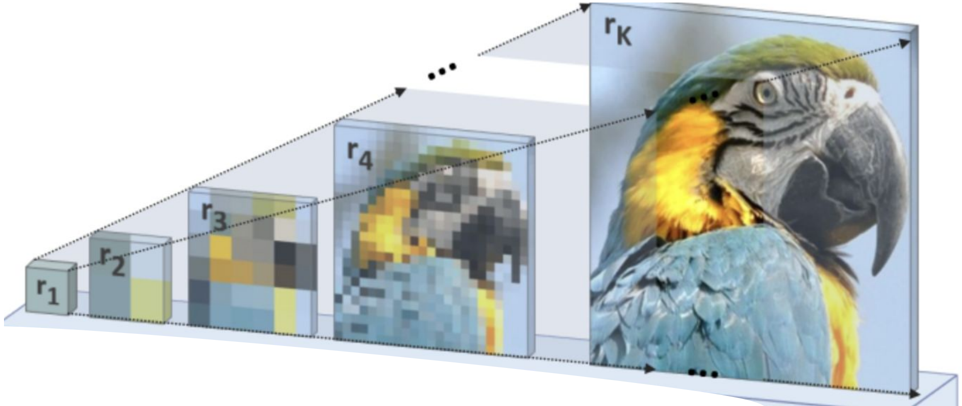
$$\mathrm{JS}(p,q) \;=\; \tfrac{1}{2}\,\mathrm{KL}\big(p\,\big\|\,m\big) \;+\; \tfrac{1}{2}\,\mathrm{KL}\big(q\,\big\|\,m\big) = H(m) - \tfrac{1}{2}H(p) - \tfrac{1}{2}H(q), \qquad m = \tfrac{p+q}{2}$$

$$W_1(p,q) = \left(\inf_{\gamma \in \Gamma(p,q)} \int d(x,y)^k \, d\gamma(x,y) \right)^{1/k} \Bigg|_{k=1} = \sup_{\|f\|_{\mathrm{Lip}} \le 1} \Big\{ \mathbb{E}_p[f] - \mathbb{E}_q[f] \Big\}$$

Diffusion Model and beyond

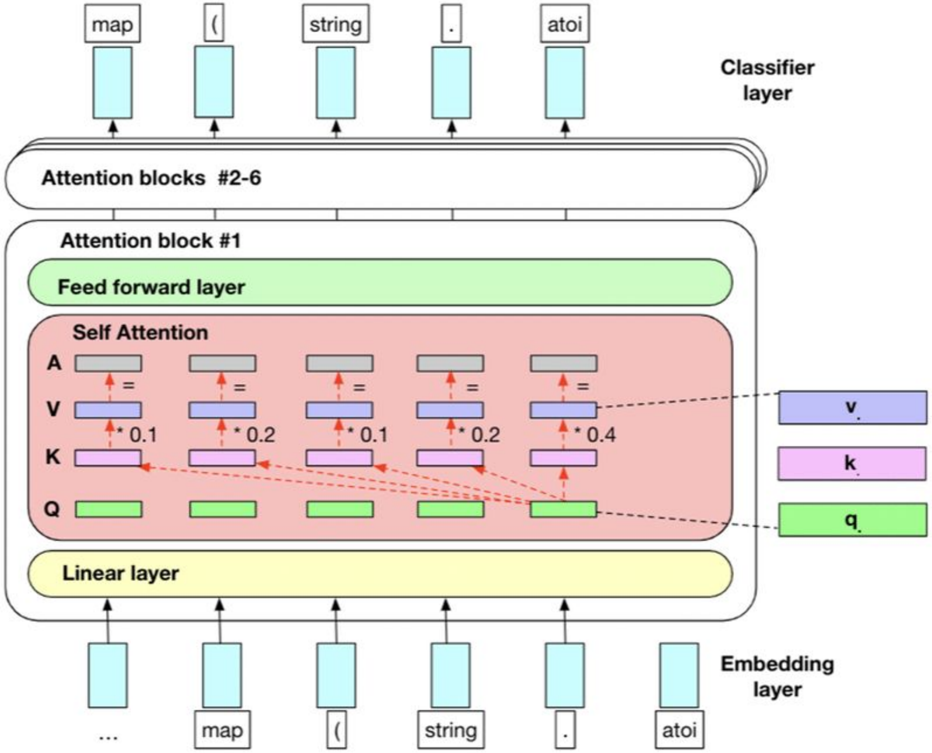
Symmetry: decompose complex distributions into simple and tractable ones

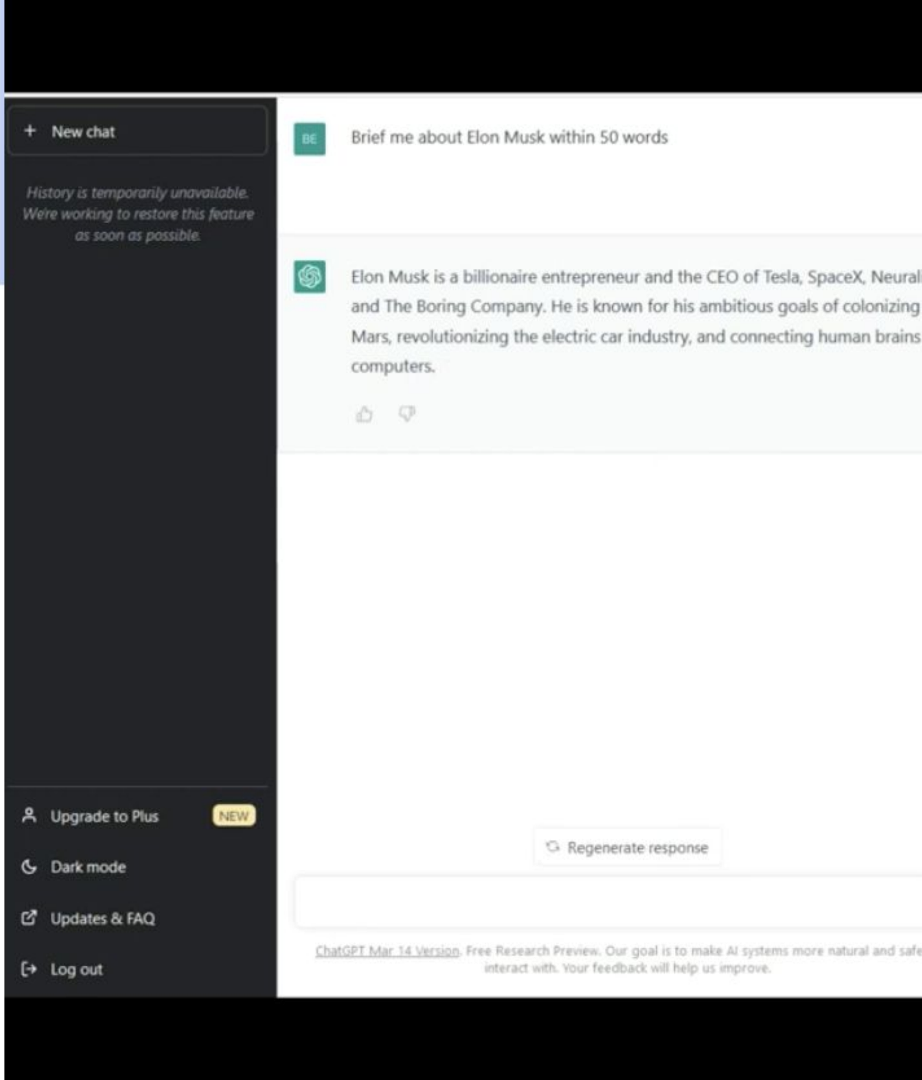




Applications

Language Generation





Audio Generation

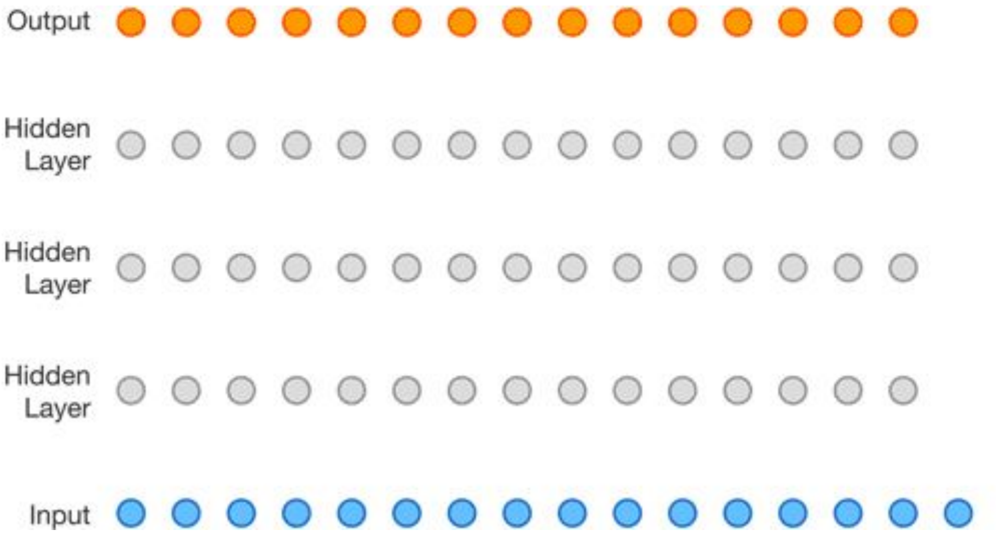
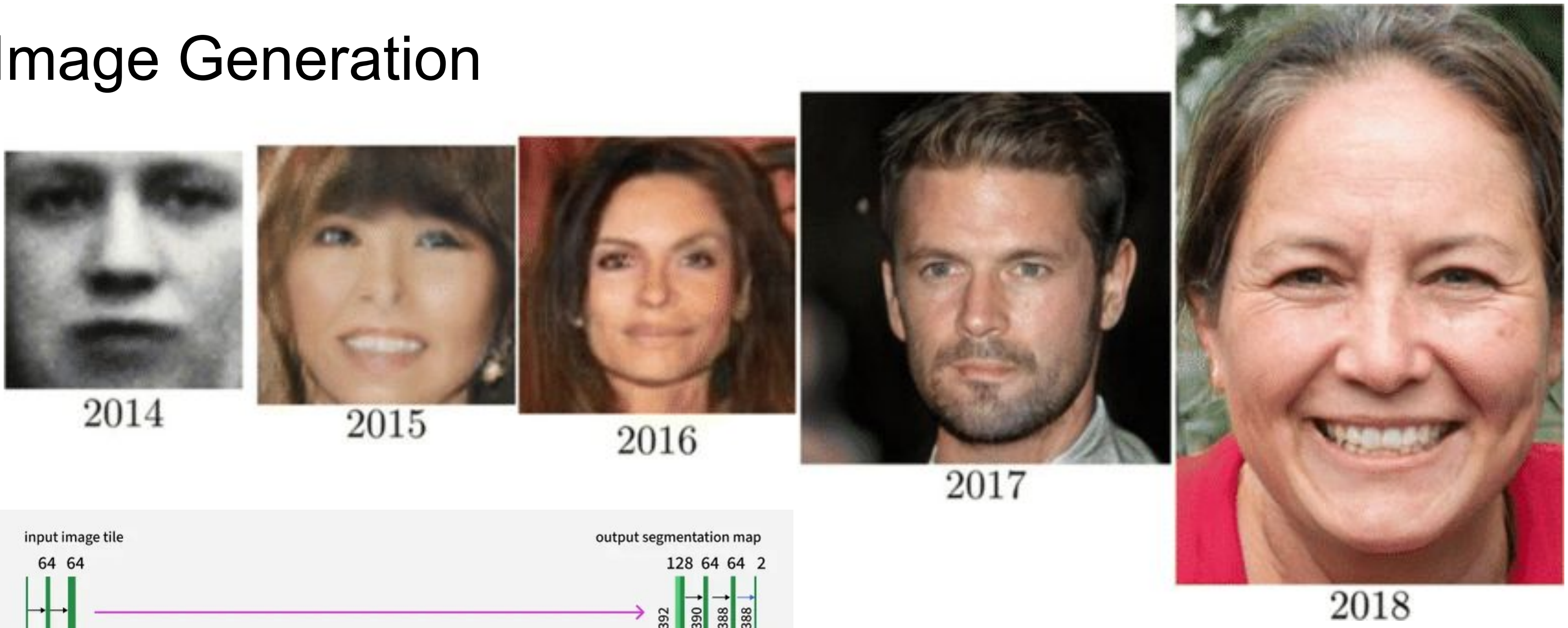
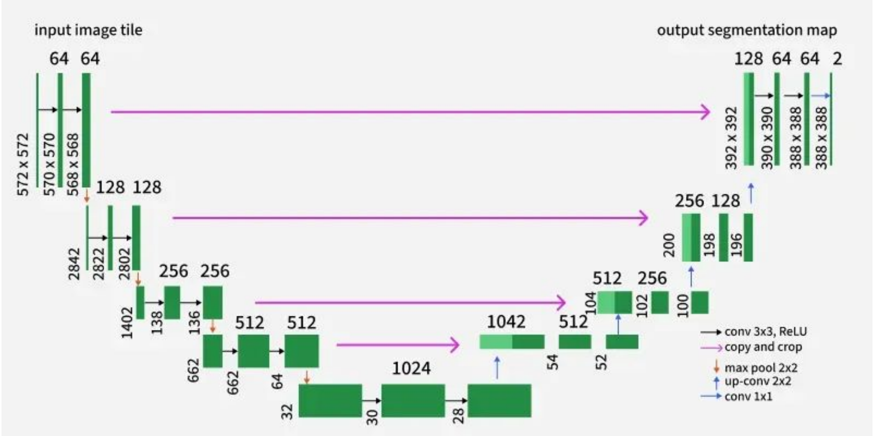
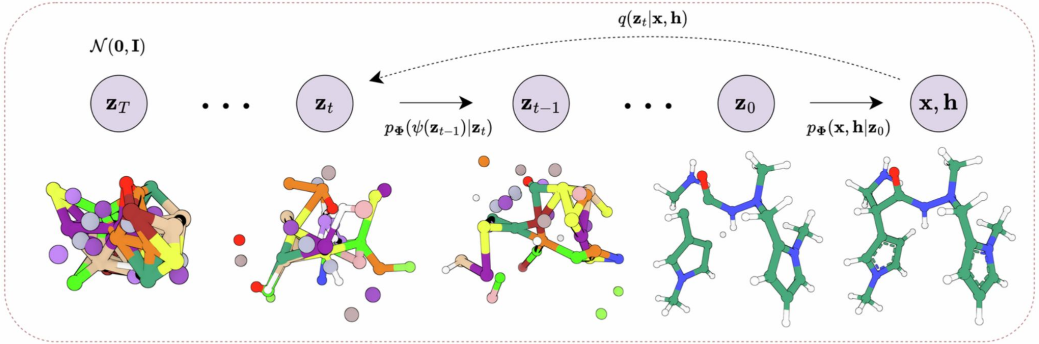


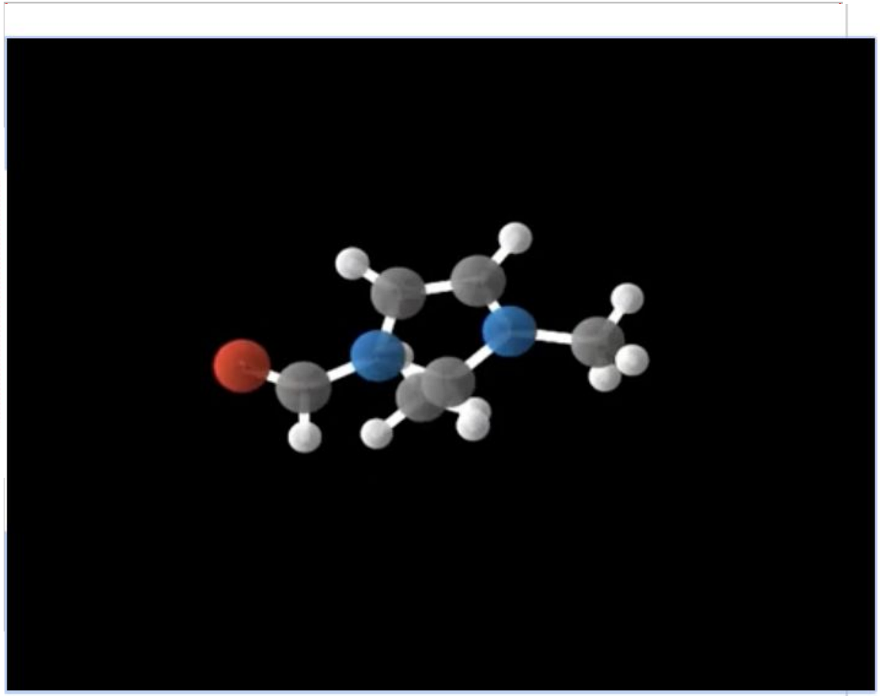
Image Generation





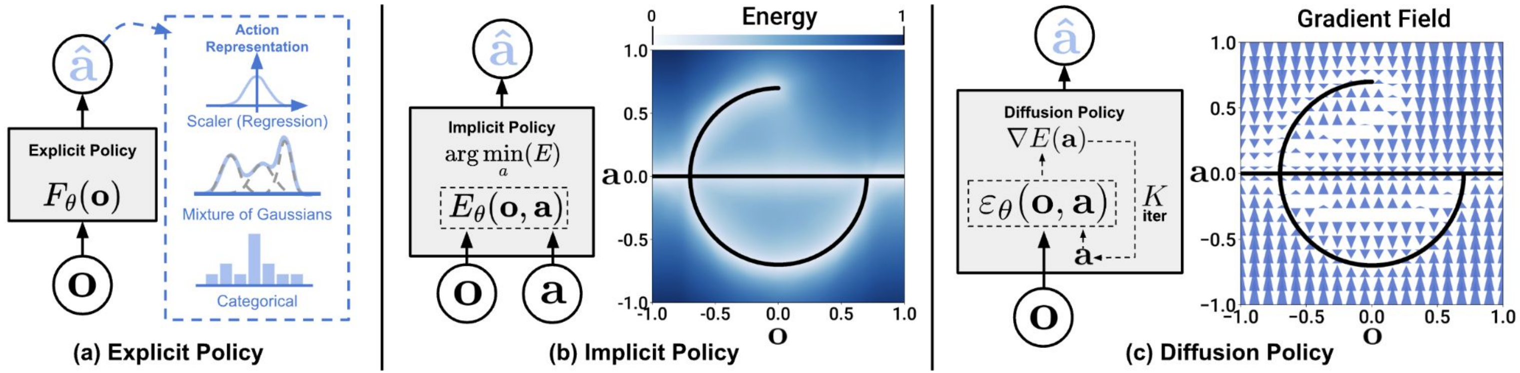
Molecule Generation

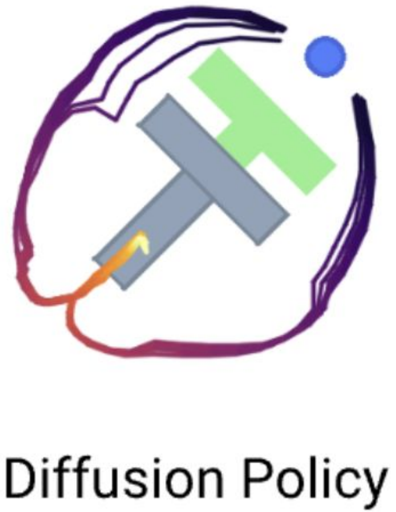


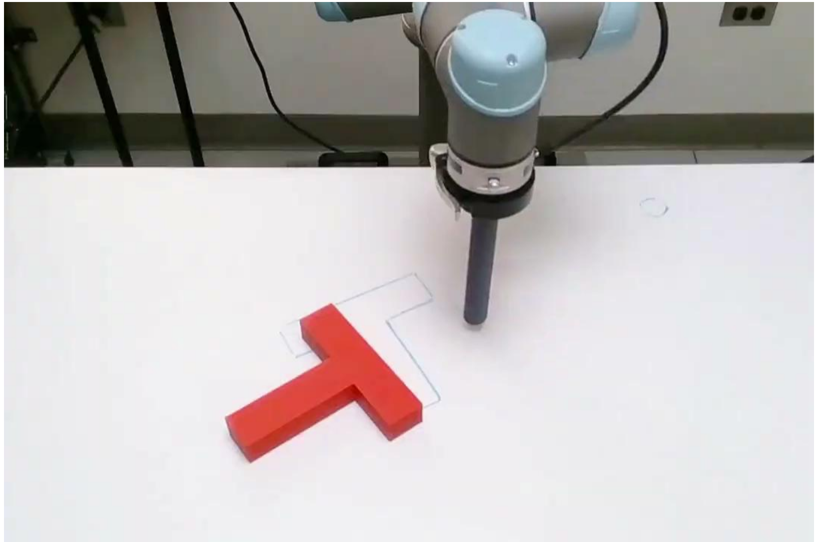


Robot Learning

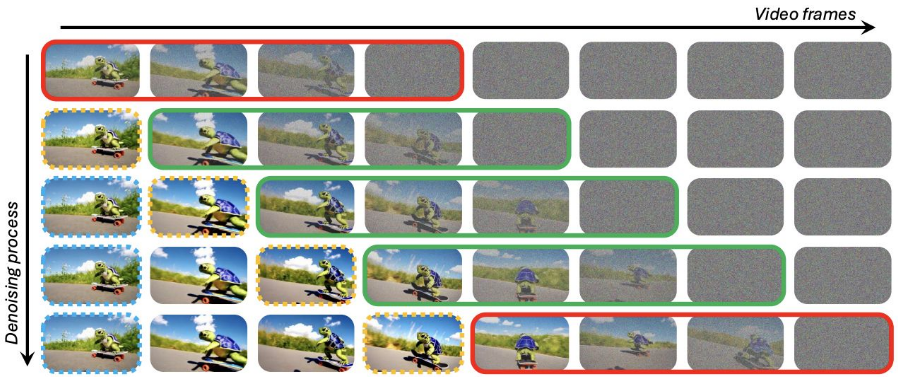
P(actions | past observations)







Video Generation



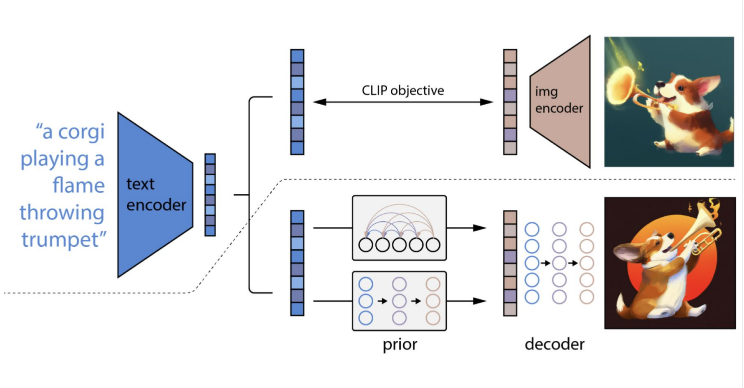
Multimodality

Example: Text to Image

User Input:

泰迪熊穿着戏服，站在太和殿前唱京剧

A teddy bear, wearing a costume, is standing in front of the Hall of Supreme Harmony and singing Beijing opera



Future Studies in Generative Modeling

Courses:

Stanford CS236 Deep Generative Models by Prof. Stefano Ermon

MIT 6.S978 Deep Generative Models by Prof. Kaiming He

To play around with generative models:

https://github.com/li-hong-yue/GenerativeModelsZoo

Thank you!

References

Stanford CS236 Deep Generative Models

MIT 6.S978 Deep Generative Models